

2023

Time - 3 hours

Full Marks - 80

*Answer all groups as per instructions.
Figures in the right hand margin indicate marks.*

GROUP - A

1. Fill in the blanks. (all) [1 × 12]
- (a) Lagrangian is a _____ function.
 - (b) The constraint of a pendulum with variable length is _____.
 - (c) $\oint \vec{F} \cdot d\vec{r} = 0$ does not hold for _____ forces.
 - (d) Shortest distance between two points in a plane is known as _____.
 - (e) The dimension of action integral of Hamilton's principle is _____.
 - (f) Fermat's principle of least time is one example of _____ principle.
 - (g) According to Minkowski, the fourth coordinate is _____.
 - (h) The square of four velocity vector is _____ invariant.

[2]

- (i) The relativistic Doppler effect holds good for _____ wave.
- (j) Dimension of Hamiltonian is equal to the dimension of _____.
- (k) Time dilation leads to the principle of _____.
- (l) For space-like interval, the square of the interval is _____ than zero.

GROUP – B

2. Answer any eight of the following questions within two to three sentences each. [2 × 8
- (a) What is the cyclic coordinate if a particle is moving in a central-force field ?
 - (b) Define Scleronomic constraints.
 - (c) A body moves with $0.2c$ velocity. Find the ratio of moving mass to rest mass.
 - (d) A rocket has velocity $0.6c$. Find the velocity of light with respect to rocket.
 - (e) Explain Minkowski space.
 - (f) Write Lorentz transformation equation.
 - (g) Prove that rest mass of photon is zero.

[3]

- (h) State principle of virtual work.
- (i) Define generalised coordinates.
- (j) Explain Lagrangian of a charged particle.

GROUP – C

3. Answer any eight of the following questions within 75 words each.
[3 × 8]

- (a) Explain the physical significance of Lagrangian.
- (b) Explain Hamilton's principle.
- (c) Explain D' Alembert's principle.
- (d) Write the rules for framing Lagrange's equation.
- (e) Explain central force motion.
- (f) Explain four velocity and acceleration.
- (g) What are time-like light-like ?
- (h) Explain Brachistochrone problem with one example.
- (i) The rest mass of electron is 0.928×10^{-31} kg. Calculate the energy equivalent in eV.
- (j) Calculate the velocity of a watch when it seems to be slowed down by 1 minute in one hour.

GROUP – D

Answer **all** questions within 500 words each.

4. Using D' Alembert's principle, derive Lagrange's equation of motion of a particle moving under the action of conservative force. [7]

OR

Using Lagrange's equation, derive the equation of motion of a one dimensional harmonic oscillator.

5. Derive Lagrange's equation of motion from Hamilton's principle. [7]

OR

Derive an expression for Hamilton's Canonical equations of motion.

6. Derive an expression for Lorentz transformation equations. [7]

OR

Using Lorentz transformation equation, derive an expression for relativistic variation of mass with velocity.

7. Derive an expression for relativistic Doppler effect using four vector concept. [7]

OR

Explain conservation of four momentum and give its application to two body decay of an unstable particle.