2023

Time - 3 hours

Full Marks - 60

*Answer **all groups** as per instructions.*
Figures in the right hand margin indicate marks.

GROUP – A

1. Answer all questions and fill in the blanks as required. [1 × 8

(a) In floating point representation, significant digits are used to express the ________ of a number accurately.

(b) Tangent method is also known as ________.

(c) Round-off error that occurs due to the approximation of ________ numbers.

(d) What is overflow and underflow ?

(e) The ________ of a numerical method indicates how quickly it converges to the solution.

(f) Write down Newton Cotes integration rules.

(g) The bisection method is used to find the ________ of a function.

(h) The Trapezoidal rule is a method for numerical ________ of a function.

GROUP – B

2. Answer any eight of the following questions within two to three sentences each. [1½ × 8

(a) What is the value of the round-off error when $\frac{1}{3}$ is approximated in a floating point representation with three significant digits ?

(b) Calculate the global truncation error after four iterations of a numerical method. Given that the local truncation error per iteration is 0.01.

(c) Define round off error in numerical computation.

(d) Compute $\sqrt{9}$ using bisection method, starting with interval [1, 4] and rounding to one decimal place.

(e) What is the region of convergence of Secant method ?

(f) Calculate the root of equation $f(x) = x^2 - 4x - 5$ using the Newton Raphson method starting $X_0 = 3$.

(g) Briefly describe the bisection method for finding the root of a function.

(h) Write down the forward difference operator.

(i) Calculate the forward difference for the data points f(1) =4, f(2) = 9 and f(3) = 16 and round the result to one decimal point.

(j) What is the purpose of the Trapezoidal rule in numerical integration ?

GROUP – C

3. Answer any eight of the following questions within 75 words each. [2 × 8

(a) Calculate the round off error when the number 0.00785 is represented with four significant digits.

(b) If the local truncation error in an iterative process is 0.02 and the process is repeated 5 times, what is the resulting global truncation error ?

(c) Using the bisection method, find the root of the function $f(x) = x^3 - 5$ within the interval [1, 2] to an accuracy of 0.01.

(d) Give examples of exact and appropriate numbers.

(e) Describe the merits of Newton's method of iterations.

(f) What is meant by linear interpolation ?

(g) Forth-order Runge-Kutta method uses how may steps ?

(h) Calculate the forward differences for the data points. f(1) = 4, f(2) = 9 and f(3) = 16 using the finite difference operators.

(i) Why is Secant method also called two point method ?

(j) Interpolate the value of y at $x - 2$ using Langrange's interpolation formula for the data point (1, 3) and (3, 9)

GROUP – D

*Answer **all** questions within 500 words each.*

4. Convert the decimal number 10.625 into its IEEE 754 single precision floating point representation. Show the binary representation. [6

OR

Explain the following concepts with suitable examples : [2 × 3

(i) Round off error

(ii) Local truncation error

(iii) Global truncation error

5. Use the bisection method to find the root of the function $f(x) = x^3 - 5x^2 + 3x + 4$ within the interval [1, 3] to an accuracy of 0.01. Show the iteration until convergence. [6

OR

Apply the Newton-Raphson method to find the root of the equation $f(x) = e^x - 3x$, starting with an initial guess of $x_0 = 1$. Provide the iterations and the final approximation at least four decimal places.

6. Using Gregory Newton forward differences, interpolate the value of y at $x = 2$ for the data points (1, 3) (3, 9) and (4, 16). Show the calculations step by step. [6

OR

Perform Linear interpolation between the data point (2, 4) and (5, 10) to estimate the value of y at $x = 3$.

7. Apply Simpson's rule to approximate the integral of the function $f(x) = x^3$ from $x = 1$ to $x = 4$ using four subintervals. Show the calculations and the final results. [6

OR

Solve the ordinary differential equation of $\frac{dy}{dx} = 2x - 3y$, where $y(0) = 1$, using the Runge-Kutta second order method over the interval of [0, 1]. Display the iterations and the solution at $x = 1$.