No. of Printed Pages : 5

5-SEMAS-Math-DSE-2(R&B)

2023

Time - 3 hours

Full Marks - 80

*Answer **all groups** as per instructions.*
Part of each question should be answered continuously.
Figures in the right hand margin indicate marks.
The symbols used have their usual meaning.

GROUP – A

1. Fill in the blanks and choose the correct answers. (all) [1 × 12

(a) If A and B are independent events, then $P(B/A) =$ ________

(b) Probability of a certain event is ________ .

(c) There are ________ number of elements in a continuous random variable.

(d) The second moment about the mean is ________ .

(e) The expectation of a constant is ________ .

(f) ________ is the value that appears most frequently in a data

(i) Mean (ii) Median

(iii) Mode (iv) Range

(g) Bernoulli distribution is a special case of ________ distribution.

(h) In negative binomial distribution ________ is fixed.

(i) Mean of standard normal distribution is ________ .

(j) The range of correlation coefficient (l) varies from ________ to ________ .

(k) If $X_1, X_2, \ldots\ldots\ldots, X_n$ are random variables, then the sample mean is ________ .

(l) The Chi-square distribution is a ________ test.

(i) parametric (ii) non-parametric

(iii) neither of the above two

GROUP – B

2. Answer any eight of the following questions. [2 × 8

(a) Prove that $P(\phi) = 0$ for any sample space S.

(b) Find the probability that a leap year has 52 Sundays.

(c) Check whether the function given by $f(x) = \frac{x+2}{25}$ for x = 1, 2, 3, 4, 5 can serve as the probability of a discrete random variable.

(d) For a random variable X, prove that

$E(aX + b) = a\,E(X) + b$ where a and b are constants.

(e) What is conditional expectation ?

(f) What is the mean and variance of exponential distribution ?

(g) Find the mean of Uniform distribution.

(h) State Central Limit theorem.

(i) What is the probability distribution of sum of 'n' independent random variables $X_1, X_2, \ldots\ldots\ldots, X_n$ having Poisson distributions with the respective parameters $\lambda_1, \lambda_2, \ldots\ldots\ldots, \lambda_n$?

(j) Define Chi-square distribution.

GROUP – C

3. Answer any eight questions [3 × 8

(a) If A and B are any two events in a sample space, then prove that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

(b) Find a formula for the probability distribution of the total number of heads obtained in four tosses of a balanced coin.

(c) A box of fuses contains 20 fuses of which 5 are defective. If 3 of the fuses are selected at random and are removed from the box in succession without replacement, what is the probability all 3 fuses are defective ?

(d) Prove that for a random variable X

$$\text{var}(aX + b) = a^2 \text{ var}(X).$$

(e) Define Bivariate Normal distribution.

(f) Define product of moments about origin.

(g) Find the mean of Beta distribution.

(h) What is 't' distribution ?

(i) If X is the number of points rolled with a balanced die, find var(X).

(j) If $X_1, X_2, \ldots\ldots\ldots\ldots, X_n$ constitute a random sample from an infinite sample with the mean μ and the variance σ^2, then prove that $\text{var}(\bar{X}) = \frac{\sigma^2}{n}$.

GROUP – D

*Answer **all** questions.*

4. State and prove Baye's theorem. [7

OR

Given the joint probability density

$$f(x, y) = \begin{cases} \frac{2}{3}(x + 2y) & \text{for } 0 < x < 1, 0 < y < 1 \\ 0 & \text{, otherwise.} \end{cases}$$

Find the conditional density of X, given Y = y and use it to evaluate $P\left(X \le \frac{1}{2} \middle| Y = \frac{1}{2}\right)$.

5. Find the mean and variance of Binomial distribution. [7

OR

Find the mean and variance of Gamma distribution.

6. State and prove Chebyshev's theorem. [7

OR

If X and Y are independent, then prove that E(XY) = E(X) . E(Y) but not conversely.

7. Given two random variables X and Y that have the joint density [7

$$f(x, y) = \begin{cases} xe^{-x(1+y)} & \text{for } x > 0 \text{ and } y > 0 \\ 0 & \text{, elsewhere.} \end{cases}$$

Find the regression equation of Y on $X = \frac{1}{x}$.

OR

If the probability density of X is given by

$$f(x) = \begin{cases} 6x(1 - x) & \text{for } 0 < x < 1 \\ 0 & \text{, elsewhere.} \end{cases}$$

Find the probability density of $Y = X^3$.