2023

Time - 3 hours

Full Marks - 80

*Answer **all groups** as per instructions.*
Part of each question should be answered continuously.
Figures in the right hand margin indicate marks.
The symbols used have their usual meaning.

GROUP – A

1. Fill in the blanks. (all) [1 × 12

(a) The general LPP with m constraints and n unknowns ($m < n$) gives _____ number of basic solutions.

(b) For maximization of LPP, the objective function coefficient for an artificial variable is _____ .

(c) The rank of a matrix $(a_{ij})_{3 \times 5}$ is less than equal to _____ .

(d) The feasible region of an LPP must be _____ set.

(e) For every _____ , there is a dual variable.

(f) If the dual has an _____ solution, primal has infeasible solution.

(g) The shadow price of a non-binding constraint is equal to _____ .

(h) An assignment problem is a special case of transportation problem, because all the rim conditions are _____.

(i) A game is said to be fair if upper value of the game = lower value of the game = _____.

(j) Game which involves more than two players are called _____.

(k) The size of pay-off matrix of a game can be reduced by _____ principle.

(l) _____ method determines an initial basic feasible solution which is very close to optimum solution in a transportation problem.

GROUP – B

2. Answer any eight of the following questions. [2 × 8

(a) What are the components of an LPP model ?

(b) Define Basic solution.

(c) Why economic interpretation of duality is necessary ?

(d) Write any two relationships between a primal and a dual.

(e) Define standard primal problem.

(f) What is a pay-off matrix ?

(g) State when Big-M method is useful.

(h) What is the difference between the balanced and unbalanced transportation problem ?

(i) Write any two applications of assignment problem.

(j) Define mixed strategy.

GROUP – C

3. Answer any eight questions [3 × 8

(a) What are the different solutions of an LPP ? Explain.

(b) Show that the following system of linear equations has a degenerate solution :

$$2x_1 + x_2 - x_3 = 2$$

$$3x_1 + 2x_2 + x_3 = 2$$

(c) What are slack and surplus variables ?

(d) Write the matrix form of a primal-dual pair.

(e) Give the mathematical formulation of a transportation problem.

(f) What are symmetrical and unsymmetrical forms of primal-dual pair ?

(g) Write the transportation problem of the assignment problem is

	A_1	A_2	A_3
R_1	1	2	3
R_2	4	5	1
R_3	2	1	4

(h) Find the Saddle point of the game :

		Player B	
		B_1	B_2
Player A	A_1	9	2
	A_2	8	6
	A_3	6	4

(i) For what value of k, the game with following pay-off matrix is strictly determinable

		Player B	
		B_1	B_2
Player A	A_1	2	6
	A_2	–2	k

(j) Write the dual of the following LPP :

Minimise $Z = 15x_1 + 10x_2$

subject to the constraints

$3x_1 + 5x_2 \geq 5$

$5x_1 + 2x_2 \geq 3$

$x_1 \geq 0 , x_2 \geq 0.$

GROUP – D

*Answer **all** questions.*

4. Use Simplex method to [7

Maximise $Z = 3x_1 + 2x_2 + 5x_3$

subject to $x_1 + 2x_2 + x_3 \leq 430$

$3x_1 + 2x_3 \leq 460$

$x_1 + 4x_3 \leq 420$

$x_1 , x_2 , x_3 \geq 0$

OR

Using Big-M method to solve :

Maximise $Z = 3x_1 + 2x_2$

subject to $2x_1 + x_2 \leq 2$

$3x_1 + 4x_2 \geq 12$

$x_1 , x_2 \geq 0$

5. State and prove Fundamental theorem of duality. [7

OR

Use duality to solve the LPP :

Maximise $Z = 40x_1 + 50x_2$

subject to $2x_1 + 3x_2 \leq 3$

$8x_1 + 4x_2 \leq 5$

$x_1, x_2 \geq 0$

6. Obtain an initial basic feasible solution to the following transportation problem using the North-West corner rule : [7

	D	E	F	G	Available
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Requirement	200	225	275	250	

OR

Solve the following assignment problem :

	I	II	III	IV
A	18	26	17	11
B	13	28	14	26
C	36	19	18	15
D	19	26	24	10

7. For the game with the following pay-off matrix [7

$$P_1 \begin{bmatrix} 5 & 1 \\ 3 & 4 \end{bmatrix} \quad P_2$$

determine the optimum strategies and value of the game.

OR

Solve the following game graphically :

$$\text{Player A} \begin{bmatrix} 3 & -3 & 4 \\ -1 & 1 & -3 \end{bmatrix} \quad \text{Player B}$$