No. of Printed Pages : 5 5-SEMAS-Math-C-11(R&B)

2023

Time - 3 hours

Full Marks - 80

*Answer **all groups** as per instructions.*

Part of each question should be answered continuously.

Figures in the right hand margin indicate marks.

The symbols used have their usual meaning.

GROUP – A

1. Answer all questions and fill in the blanks as required. [1 × 12

   (a) Find the partial derivatives of $f(x, y) = 2xy + x^2$ with respect to x at (1, 2) is ________.

   (b) $\lim\limits_{(x, y) \to (1, 2)} \dfrac{2x^2y}{x^2 + y^2}$ = ________.

   (c) Given $\lim\limits_{(x, y) \to (1, 1)} f(x, y) = 5$. If f is continuous, then the value of $f(1, 1)$ = ________.

   (d) If u is a function of two variables x and y, then the total derivatives of u is defined as ________.

   (e) The necessary condition for f(x, y) to have extreme points at (a, b) is ________.

P.T.O.

(f) Grad ϕ defines a ________ field.

(g) Evaluate $\int_0^1 \int_{-1}^1 \int_2^3 dx\, dy\, dz$.

(h) Is the vector $F = 2xyi + x^2j$ conservative with scalar potential $f = x^2y$?

(i) Write the cylindrical polar coordinate of (x, y, z).

(j) A field is conservative if ________ .

(k) Write the vector form of Green's theorem.

(l) For a plane lamina, total mass of the lamina is ________ .

GROUP – B

2. Answer any eight of the following questions. [2 × 8

(a) State Stoke's theorem.

(b) Define curl of the vector field on R^3.

(c) Find the mass of the solid at P(x, y, z) and density P = f(x, y, z).

(d) Define directional derivatives of a function.

(e) Find the repeated limits of

$$f(x, y) = \frac{xy}{x^2 + y^2} \text{ at } (x, y) = (0, 0).$$

(f) Find f_{xy} for the function $f(x, y) = x^3y + y^2$.

(g) Find $\nabla f(x, y)$ for $f(x, y) = x^2y + y^3$.

(h) Find the vector that is normal to the level surface $x^2 + 2xy - yz + 3z^2 = 7$ at the point $P_0(1, 1, -1)$.

(i) Evaluate $\int_0^2 \int_0^1 (x^2 + xy + y^2)dy\, dx$.

(j) If a is a constant vector, then find div(r × a).

GROUP – C

3. Answer any eight questions [3 × 8

(a) Verify the continuity of the function

$$f(x, y) = \begin{cases} \dfrac{x^2y}{x^3 + y^3} & , (x, y) \neq (0, 0) \\ 0 & , (x, y) = (0, 0) \end{cases} \text{ at } (0, 0).$$

(b) Find the directional derivative of the function $f(x, y, z) = x^2y - yz^3 + z$ at the point (1, –2, 0) in the direction of the vector $a = 2i + j - 2k$.

(c) Find an equation for the tangent plane and parametric equations for the normal line to the surface $z = x^2y$ at the point (2, 1, 4).

(d) Find the absolute maximum and minimum values of $f(x, y) = 3xy - 6x - 3y + 7$ on the closed triangular region R with vertices (0, 0), (3, 0) and (0, 5)

(e) Find all relative extrema and Saddle points of the function

$$f(x, y) = 2x^2 + 2xy + y^2 - 2x - 2y + 5.$$

(f) Find the work done when a force

$$F(x^2 - y^2 + x)i - (2xy + y)j$$

moves a particle in the plane from (0, 0) to (1, 1).

(g) Evaluate the surface integral $\iint_C xz\, ds$ where C is the part of the plane $x + y + z = 1$ which lies in the first octant.

(h) Evaluate $\int_0^a \int_0^b \int_0^c x^2y^2z^2\, dx\, dy\, dz.$

(i) Write the fundamental theorem of Line integral.

(j) Calculate the volume of the solid bounded by the planes $x = 0$, $y = 0$, $x + y + z = a$ and $z = 0$.

GROUP – D

*Answer **all** questions.*

4. Find the equation of the tangent plane to the surface [7

$$x^3 + y^3 + 3xyz = 3 \text{ at } (1, 2, -1).$$

OR

If $f(x, y) = \begin{cases} \dfrac{xy}{x^4 + y^4} & , x \neq 0, y \neq 0 \\ 0 & , x = 0, y = 0, \end{cases}$

show that f_x, f_y exist at (0, 0) but f(x, y) is discontinuous at (0, 0).

5. Show that the function $f(x, y) = 2x^4 - 3x^2y + y^2$ has neither maximum nor minimum at (0, 0). [7

OR

Prove that $\nabla(u \times v) = (\nabla . u)v - (\nabla . v)u$.

6. Evaluate $\int_{-1}^{1} \int_{0}^{2} \int_{x-2}^{x+2} (x + y + z)\, dx\, dy\, dz.$ [7

OR

Find the volume of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

7. Verify Stoke's theorem for $F = x^2i + xyj$ taken over the square in the plane $z = 0$ whose sides are along the lines $x = 0$, $y = 0$, $x = a$, $y = a$. [7

OR

State and prove Green's theorem.