No. of Printed Pages : 4 3-SEMS-Phy-CC-V(R&B)

2023

Time - 3 hours

Full Marks - 60

*Answer **all groups** as per instructions.*

Figures in the right hand margin indicate marks.

GROUP – A

1. Fill in the blanks. (all) [1 × 8

(a) The period of Cosine function is ________.

(b) The quotient of two odd functions is a ________ function.

(c) For a periodic function f(x) with time period T, $f(x + T) =$ ________.

(d) The value of Legendre's polynomial $P_0(x)$ is ________.

(e) The value of Bessel's polynomial $J_0(0)$ is ________.

(f) The differential equation $y'' - 2xy'' + 2ny = 0$ is known as differential equation.

(g) The value of $\Gamma(0)$ is ________.

(h) $\text{erf}(x) + \text{erf}(-x) =$ ________.

GROUP – B

2. Answer any eight of the following questions within two to three sentences each. [1½ × 8

(a) Prove that $\Gamma(-2) = \infty$.

(b) Write complex form of the Fourier series.

(c) Explain regular singular point.

(d) Write Fourier sine series.

(e) Prove that $\int_0^\infty \frac{x^8(1 - x^6)}{(1 + x)24} dx = 0$.

(f) Find the value of $\beta\left(\frac{5}{2}, \frac{3}{2}\right)$.

(g) Prove that $\Gamma(1) = 1$.

(h) Prove that product of two odd functions is an even function.

(i) Write two important properties of Hermite polynomials.

(j) Prove that $P_0(x)$ and $P_1(x)$ are orthogonal to each other.

GROUP – C

3. Answer any eight of the following questions within 75 words each. [2 × 8

(a) Show that derivative of any even function is an odd function.

(b) Write Dirichlet condition.

(c) Prove that $\Gamma(n + 1) = n\Gamma(n)$.

(d) Write Laplace's equation in Cartesian coordinates.

(e) Find the period of the periodic function

$$f(t) = A \sin\left(\frac{t}{T} + \frac{\pi}{4}\right)$$

(f) Write Bessel's differential equation.

(g) State orthogonality relation of Legendre's polynomial.

(h) Find the value of $H_2(x)$.

(i) Write solution of Laplace equation in spherical symmetry.

(j) Prove that $\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right) = \sqrt{2}\pi$.

GROUP – D

*Answer **all** questions.*

4. Prove that [6

$$\int_0^{\frac{\pi}{2}} \sin^p\theta \, . \cos^q\theta \, d\theta = \frac{\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(\frac{q+1}{2}\right)}{2\Gamma\left(\frac{p+q+2}{2}\right)}$$

Hence prove that $\int_0^{\frac{\pi}{2}} \sin\theta \, d\theta = 1$.

OR

(a) Prove that $2x\,H_n(x) = 2n\,H_{n-1}(x) + H_{n+1}(x)$. [3

(b) Prove that $\int_{-1}^{1} x^3 P_3(x)dx = \frac{4}{35}$. [3

5. Find the Fourier series expansion of $f(x) = x^2, -\pi < x < \pi$. [6

OR

Find the Fourier series of $f(x) = x^2 + x$ in the interval $[-\pi, \pi]$.

6. Solve $x^2y'' + xy' + (x^2 - 1)y = 0$. [6

OR

Using the generating function for $H_n(x)$, derive Rodrigues formula.

7. Deduce the radial equation for the problems of spherical symmetry from Laplace's equation by variable separation method. [6

OR

Apply Laplace's equation to discuss dielectric sphere problem in an exeter uniform electric field.