

2023

Time - 3 hours

Full Marks - 60

Answer all groups as per instructions.

Figures in the right hand margin indicate marks.

GROUP – A

1. Answer all questions and fill in the blanks as required. [1 × 8]
- (a) What is meant by dual of a proposition ?
 - (b) Construct a truth table for the logical operator NAND.
 - (c) $\neg \forall x P(x) \equiv$ _____ ?
 - (d) How many bit strings of length n contain exactly r 1's ?
 - (e) $P(6, 3) =$ _____.
 - (f) Define recurrence relation.
 - (g) A vertex is _____ if it has degree 1.
 - (h) Which is stronger : NFA or DFA ?

[2]

GROUP – B

2. Answer any eight of the following questions within two to three sentences each. [1½ × 8]

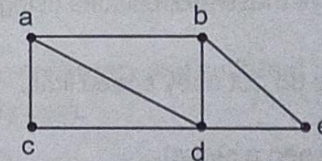
- (a) Use a truth table to verify $P \wedge T \equiv P$.
- (b) Let $P(x)$ be the statement " $x + 1 > x$ ". What is the truth value of the quantification $\forall x P(x)$, where the universe of discourse consists of all real numbers ?
- (c) Define an onto / surjective function.
- (d) How many permutations of the letters A B C D E F G H contain the string ABC ?
- (e) How many strings of length n can be formed from an English Alphabet ?
- (f) Find the first five terms of the sequence defined by the following recurrence relation and initial condition $a_n = 6a_{n-1}$, $a_0 = 2$.
- (g) Is the recurrence relation $H_n = 2H_{n-1} + 1$ homogeneous ? Why or why not ?
- (h) Define pseudograph.
- (i) Give the formal definition of a context free graph.
- (j) Define alphabet, string, language.

[3]

GROUP – C

3. Answer any eight of the following questions within 75 words each. [2 × 8]

- (a) What are the negation of the statements :
"There is an honest politician."
"All Americans eat cheese burger."
- (b) Use mathematical induction to prove that $n < 2^n$.
- (c) What is a proper subset ?
- (d) How many ways are there to select a 1st, 2nd and 3rd prize winner from 100 different people who have entered a contest ?
- (e) A person deposits ₹ 1,000 in an account that yields 9% interest compounded yearly. Set up a recurrence relation for the amount in the account at the end of n years.
- (f) State the handshaking theorem.
- (g) Define in-degree and out-degree of a vertex v .
- (h) Does the following graph have an Euler circuit or Euler path ?



- (i) What is meant by a lattice ?
- (j) Give an example of a bipartite graph.

[4]

GROUP – D

Answer all questions.

4. (a) Translate each of these statements into logical expression using predicates and quantifiers : [3]

- (i) No one is perfect.
- (ii) Not everyone is perfect.
- (iii) All your friends are perfect.

(b) Determine the truth value of each of these statements if the universe of discourse consists of all integers : [3]

- (i) $\forall n(n + 1 > n)$
- (ii) $\exists n(2_n = 3_n)$
- (iii) $\forall n(n^2 \geq n)$

OR

(a) Explain negation of these propositions using quantifiers : [3]

- (i) Some drivers do not obey speed limit.
- (ii) No one can keep a secret.
- (ii) All Bollywood movies are comedy.

[5]

(b) Determine whether $\forall x(P(x) \rightarrow Q(x))$ and $\forall x P(x) \rightarrow \forall x Q(x)$ have same truth value. [3]

A coin is flipped 8 times where each flip comes up either head or tail. How many possible outcomes : [6]

- (i) are there in total ?
- (ii) contains exactly three heads.
- (iii) contains atleast three heads.

OR

Find a solution to the recurrence relation $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$ with initial condition $a_0 = 2, a_1 = 5, a_2 = 15$.

- 5. (a) Prove that an undirected graph has an even number of vertices of odd degree. [3]
- (b) Show how graphs can be represented using adjacency lists and adjacency matrices. [3]

OR

- (a) State the necessary and sufficient conditions for Euler circuit and Euler path. [4]
- (b) How many edges must be removed from a connected graph with n vertices and m edges to produce a spanning tree. [2]

[6]

7. Differentiate between DFA and NFA.

[6

OR

State pumping lemma for regular languages.

Prove that $L = \{0^n 1^n \mid n \geq 0\}$ is not regular using pumping lemma.