No. of Printed Pages : 5

2023

Time - 3 hours

Full Marks - 60

*Answer **all groups** as per instructions.*
Part of each question should be answered continuously.
Figures in the right hand margin indicate marks.
The symbols used have their usual meaning.

GROUP – A

1. Answer all questions and fill in the blanks as required. [1 × 8

   (a) Give an example of a first order non-linear partial differential equation.

   (b) The most general quasi-linear partial differential equation must of the form __________.

   (c) How many independent variables have a partial differential equation ?

   (d) If equation is elliptic, then $B^2 - 4AC$ is __________.

   (e) The wave equation is __________ type.
   (hyperbolic, parabolic, elliptic)
   (Choose the correct answer.)

(f) Cauchy problem deals with ________ order partial differential equation.

(g) If $u(x, t) = U\left(t - \frac{x}{c}\right) H\left(t - \frac{x}{c}\right)$,

then H is known as ________ step function.

(h) The system $(a_1 - m)A + b_1B = 0$

$$a_2A + (b_2 - m)B = 0$$

has the trivial solution if ________.

(Choose the correct answer.)

(i) $A = B = 0$ (ii) $A \neq 0$ and $B = 0$

(iii) $A = 0$ and $B \neq 0$

GROUP – B

2. Answer any eight of the following questions. [1½ × 8

(a) Define a quasi-linear partial differential equation.

(b) If $u(x, y) = f(x) . g(y)$, then what happened to the equation

$$u_{xx} + u_{yy} = 0.$$

(c) Eliminate the arbitrary constant from

$$z = ax + by + a^2 + b^2.$$

(d) Classify the equation $u_{xx} - y^4u_{yy} = 2y^3u_y$.

(e) Write the Laplace equation in polar coordinates.

(f) What is D' Alembert's solution of the Cauchy problem.

(g) Write the Cauchy problem for the non-homogeneous wave equation.

(h) Write the normal form in case of linear system of three differential equations.

(i) If m_1 and m_2 are the two complex conjugate roots, then what can be the solution ?

(j) State the existence and uniqueness of the solution of the system in normal form.

GROUP – C

3. Answer any eight questions [2 × 8

(a) Solve : $u_{xy} = 4xe^{2y}$.

(b) Eliminate the function f from the relation $z = e^{mx} f(x + y)$.

(c) Solve : $au_x + bu_y = 0$.

(d) Solve the Cauchy problem :

$$uu_x + u_y = u, \ u(x, 0) = 2x, \ 1 \leq x \leq 2.$$

(e) Solve : $\frac{\partial u}{\partial x} = 4 \frac{\partial u}{\partial y}$ with $u(0, y) = 8e^{-3y}$.

(f) Show that $u(x, t) = \frac{c}{\sqrt{t}} e^{\left(\frac{-x^2}{4kt}\right)}$ is a solution of $u_t - ku_{xx} = 0$.

(g) Determine the solution of initial value problem

$$u_{tt} - c^2 u_{xx} = 0,\ u(x, 0) = 0,\ u_t(x, 0) = 3.$$

(h) Find the longitudinal oscillation of a rod subject to the initial conditions $u(x, 0) = \sin x$, $u_t(x, 0) = x$.

(i) Show that the two solutions $x = 3e^{7t}$, $y = 2e^{7t}$ and $x = e^{-t}$, $y = -2e^{-t}$ are linearly independent on every interval [a, b] and find the general solution.

(j) Show that the ordered pair of functions defined for all t by $(3e^{7t}, 2e^{7t})$ is a solution of the system

$$\frac{dx}{dt} = 5x + 3y$$

$$\frac{dy}{dt} = 4x + y.$$

GROUP – D

*Answer **all** questions.*

4. Reduce the equation $u_x - yu_y - u = 1$ into canonical form and find the general solution. [6

OR

Find the general solution of the equation

$$(y + u)u_x + (x + u)u_y = x + y.$$

5. Find the solution of the wave equation $u_{tt} - c^2 u_{xx} = 0$. [6

OR

Classify the equation $u_{xx} + 10u_{xy} + 9u_{yy} = y$, then find its general solution.

6. Determine the solution of the following initial boundary value problem : [6

$$u_{tt} = c^2 u_{xx},\ 0 < x < \pi,\ t > 0,\ u(x, 0) = 0$$

$$u_t(x, 0) = 8\sin^2 x,\ 0 \le x \le \pi,\ u(0, t) = 0 = u(\pi, t),\ t > 0.$$

OR

Find the solution of the vibrating string problem :

$$u_{tt} = c^2 u_{xx},\ 0 < x < 1,\ t > 0,\ u(x, 0) = x(1 - x)$$

$$u_t(x, 0) = 0,\ 0 \le x \le 1,\ u(0, t) = u(1, t) = 0,\ t > 0.$$

7. Find the general solution of the linear system : [6

$$2\frac{dx}{dt} + \frac{dy}{dt} - x - y = e^{-t}$$

$$\frac{dx}{dt} + \frac{dy}{dt} + 2x + y = e^{t}.$$

OR

Consider the initial value problem $\frac{dy}{dx} = 1 + y^2$. Apply method of successive approximations, starting with zeroth approximation $\phi_0(x) = 0$, compute $\phi_4(x)$.